AlternativeSoft
Analytical Investment Solutions

5 Harbour Exchange
Canary Wharf
London
E14 9GE

www.alternativesoft.com
information@alternativesoft.com
+44(0)20 7510 2003
+44(0)7813 609 835

Black-Litterman based portfolio optimization

Krishna Nehra, and Laurent Favre

Diversification has been considered a key to managing risk in an uncertain financial market. In the fund of funds space the allocation of capital across many underlying managers and strategies helps in reducing overall portfolio volatility, thus leading the funds of funds (who practice naive diversification) to decrease their portfolio volatility but increase their market beta. Traditional portfolio construction techniques solely rely on past data because of the unavailability of future fund statistics. However it is possible that investors hold some private information about one/several funds as a consequence of their skills and efforts, and they want to incorporate their views while constructing their portfolios.

Investors can express their views in the form of either explicit (absolute) funds' returns or relative returns of two or more funds. Converting these views into a portfolio return forecast vector is another challenge in addition to achieving diversification. Both of the aforementioned challenges are addressed by the well-known Black-Litterman (BL) model which forecasts the portfolio return vector by combining the investors' views and the funds' market returns. If an investor does not have any view on the returns of his funds then the BL model uses the market portfolio [1] as the forecast of the investors' portfolio returns. Optimization using the return vector forecasted by the BL model produces a well-diversified portfolio on the efficient frontier.

AlternativeSoft has introduced a set of fully-automated BL based optimization techniques which also have the ability to use non-normally distributed fund returns. In addition, AlternativeSoft's software platform provides an easy to use interface to let the users define their own views about the assets' expected returns and their confidence (three levels - High, Medium, Low) in these views. The portfolios obtained by employing these optimization techniques have: (i) low volatility (Mean-Variance optimization), or (ii) small average extreme losses (Mean-Conditional VaR optimization), or (iii) low volatility, high skewness, and low kurtosis (Mean-Modified VaR optimization). This article presents an example of portfolios optimized by AlternativeSoft's BL model based optimization with the following objectives:

- Minimize the portfolio volatility,
- Produce a well-diversified portfolio,
- Allow the investors to define their views (absolute as well as relative) on the funds' (mutual funds and hedge funds) expected returns,
- Tilt the portfolio from an equally weighted portfolio towards the funds with good future expected returns if the investors are confident about the future returns of these funds.

Data and optimization steps

We want to construct optimal portfolios of 6 funds given in Table 1 under two scenarios: (1) Investors hold no view on the funds' returns, (2) Investors hold one absolute and one relative view on the funds' returns.[3] Columns 'Investor's Expected Returns' & 'Confidence' respectively show the investor's views on two assets' expected returns and his confidence in those. Columns 'BL Equilibrium Return' & 'BL Combined Return' are derived from the BL model in

[1] Market portfolio is the portfolio based on the funds' market capitalization weights.

[3] The statistics used in the table are based on real funds but their names have been changed to keep anonymity.

Table 1: **Annualized return vectors**

Funds	Investor's Expected Returns	Confidence	BL Equilibrium Return	BL Combined Return
Equity Hedge 1			2.10%	0.24%
Equity Hedge 2	(Equity Hedge 2 - Macro 2 = 2%)	High	18.12%	11.55%
Equity Hedge 3			6.53%	5.76%
Event-Driven			5.78%	6.71%
Macro 1			0.56%	1.58%
Macro 2	9.53%	High	3.10%	9.52%

[2]Source: Alternativesoft. Data from August 2007 to May 2012.

three steps - calculate equilibrium fund returns using reverse optimization, define investors' expected returns (views), and combine these expected returns with the equilibrium returns. The following subsections deal with the two specific scenarios mentioned earlier.

Scenario 1

When an investor does not hold any view on the funds' expected returns then it is very easy for him to build a smooth efficient frontier of well-diversified portfolios using AlternativeSoft's automated BL model based optimization in just one click. In this case the BL model considers the return vector obtained from reverse optimization of the equally-weighted portfolio as the forecasted portfolio return vector. It should be noted that the original BL model uses the market portfolio for reverse optimization but the literature suggests that the equally-weighted portfolio can also achieve a consistent risk-adjusted performance, see reference [SJSW08].

Reverse optimization

We assign equal weights to all constituent funds of the portfolio and use those weights as equilibrium weights.[4] The equilibrium returns (Π) can be expressed as

$$\Pi = \delta \mathbf{\Sigma} \mathbf{w}, \tag{1}$$

where

$\mathbf{\Sigma}$ is the covariance matrix of the fund returns,
$\mathbf{w}$ is the vector of equilibrium weights, and
δ is risk aversion coefficient.
Column 'BL Equilibrium Returns' in Table 1 shows the equilibrium returns of our example portfolio calculated using Equation (1). These are the future fund returns if the portfolio was equally weighted.

Figure 1 exhibits the portfolio weights on the efficient frontier for Scenario 1 computed by AlternativeSoft's BL-based mean-variance optimization technique. It can be seen that the obtained optimal portfolios are highly diversified.

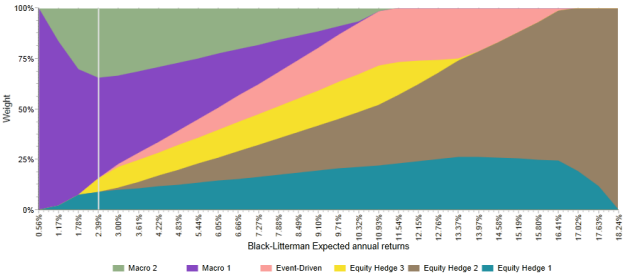

Figure 1: Distribution of weights when the investor has no view on funds' expected returns, Min weight = 0%, Max weight = 100%

Scenario 2

Here we assume that an investor has two views on two assets' expected returns in Table 1 and that he has a high confidence in these expected funds' returns. These views are defined below, where view 1 is an

[4]Another technique to determine equilibrium weights is given in [He99].

absolute view and View 2 is a relative view.

View 1: Macro 2 will have an absolute return of 9.53% (Confidence in this expected return = High).

View 2: Equity Hedge 2 will outperform Macro 2 by 2% (confidence in this expected return = High)

Combine expected returns with equilibrium returns

Now we have the equilibrium returns (i.e. when the investor has no view on the funds' expected returns) and the investors' views on the expected returns of two funds, so we compute a weighted combination of these inputs to obtain the BL combined return column (see column 'BL Combined Return' of Table 1).[5] The highlighted cells in column 5 of Table 1 show that the investor's high confidence in his two views has tilted the BL combined return vector towards these views.

Figure 2 displays the portfolio weights on the efficient frontier for scenario 2. Portfolio weights for Scenario 2 are less diversified compared to those for Scenario 1, which can be explained by the fact that Scenario 2 has also taken into account that the investor has a high confidence in the expected returns of the funds representing the green and purple shaded areas. One sees that as soon as the investor is confident in the expected return of a fund, the fund's weight is automatically increased within his portfolio. Thanks to this BL based optimization technique, errors due to wrong funds' expected returns are diminished and portfolios should outperform benchmarks.

Conclusion

This article presented a Black-Litterman based optimization technique to obtain well diversified, intuitive, and stable out-of-sample portfolios. However the technique proposed in this paper is not limited to volatility minimization portfolios, it can be used

[5]For more details and mathematical derivation of the BL combined return column refer [He99, FFK10, Idz02].

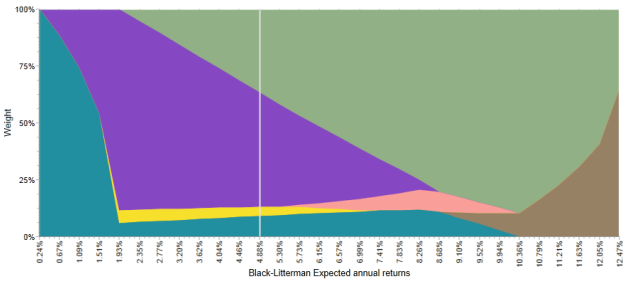

Figure 2: Distribution of weights when the investor has views on absolute expected return of Macro2 (in green shaded area) and on relative expected return of Macro2 and Equity Hedge 2(purple shaded area)

to obtain portfolios with low volatility, high skewness, and low kurtosis. In addition, the technique can help to tilt the portfolio from equally weighted space towards funds with good expected performance if investors are certain about their performance. This is all done in a quick and easy to use manner - a benchmark which stands throughout AlternativeSoft.

References

[FFK10] F.J. Fabozzi, S.M. Focardi, and P.N. Kolm, *Quantitative equity investing: Techniques and strategies*, Wiley, 2010.

[He99] G. He, *The intuition behind black-litterman model portfolios*, Investment Management Research (1999).

[Idz02] T.M. Idzorek, *A step-by-step guide to the black-litterman model*, Forecasting expected returns in the financial markets (2002), 17.

[SJSW08] J.S. Scott, C.L. Jones, J.W. Shearer, and J.G. Watson, *Enhancing utility and diversifying model risk in a portfolio optimization framework*, January 22 2008, US Patent 7,321,871.